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Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Tunnel-Induced Gradients and Their Effect on Drag"

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MOKRY[1] has commented on an analysis by Hackett[2] that leads to a wake-induced drag increment that is independent of the model volume for low-speed flows in solid-wall wind tunnels. In this respect, Hackett's analysis contradicts the classical result given in Ref. 3, recently also obtained by Taylor[4] using an approach similar to that used by Mokry.

The object of this Comment is to show how it is possible to recover the classical result using the form of analysis given by Taylor. Taylor's analysis was performed for compressible flows, but to allow a comparison with Hackett's and Mokry's analysis, the flow will be assumed to be incompressible.

The difference between the method of Taylor and that of Mokry is that the former considers only the flow outside the wake, whereas the latter includes the contribution of the base pressure on the drag of the displacement surface representing the wake of the model. Thus Taylor's method includes the effect of the change in pressure forces on the displacement surface due to constraint, whereas Mokry implicitly assumes that it is exactly countered by the base-pressure drag, which is only true in unconstrained flow (d'Alembert's paradox[5]). The notation used here is as before.[1] To aid the comparison of results, U is taken to be axial velocity, ρ is density, p is pressure, and C and Q are the cross-sectional areas of the test section and the wake. Conservation principles are applied to a box containing the (parallel) walls of the wind tunnel and two stations, one far upstream (station 1) and the other far downstream (station 2). Thus the momentum equation given by Mokry,

$$C\left(\rho U_1^2 + p_1\right) = (C - Q)\rho U_2^2 + Cp_2 + F$$

is replaced by

$$C\left(\rho U_1^2 + p_1\right) = (C - Q)\left(\rho U_2^2 + p_2\right) + F \tag{1}$$

The mass and energy conservation equations remain unchanged, namely

$$CU_1 = (C - Q)U_2 \tag{2}$$

$$\tfrac{1}{2}\rho U_1^2 + p_1 = \tfrac{1}{2}\rho U_2^2 + p_2 \tag{3}$$

Thus combining Eqs. (1–3), it follows that

$$F = Qp_2 + \tfrac{1}{2}\rho U_1^2 C[Q/(C - Q)]^2 \tag{4}$$

Consider now an equivalent unconstrained free-air flow with the same wake displacement area, for which the freestream static pressure is p_c, i.e., the corrected tunnel static pressure. For this flow, there is obtained the result

$$F_c = Qp_c$$

as can readily be inferred from Eq. (4) by replacing suffix 2 with suffix c and taking the limit as C becomes infinitely large compared with Q. Therefore the correction to drag is

$$\Delta D = (p_c - p_2)Q - \tfrac{1}{2}\rho U_1^2 C[Q/(C - Q)]^2 \tag{5}$$

The first term on the right-hand side is the drag correction obtained by Taylor after neglecting "higher-order" terms in the compressible Bernoulli equation, and the second term is that obtained independently by Hackett[2] and Mokry.[1]

Write

$$p_c - p_2 = p_c - p_1 + p_1 - p_2 \tag{6}$$

and note that

$$p_c - p_1 = -\rho U_1^2 \varepsilon \tag{7}$$

to the order of accuracy of linear theory,[4] where ε is the nondimensional blockage increment in velocity. Then, observing from Eq. (3) that

$$p_1 - p_2 = \tfrac{1}{2}\rho U_1^2\{[C/(C - Q)]^2 - 1\} \tag{8}$$

and combining Eqs. (6–8) with Eq. (5), there is obtained the result

$$\Delta D = -\rho U_1^2 Q\left[\varepsilon - \tfrac{1}{2}(Q/C)\right] \tag{9}$$

ignoring terms of $O(Q/C)^2$. Because

$$\varepsilon_w = \tfrac{1}{2}(Q/C)$$

Eq. (9) reduces to

$$\begin{aligned}\Delta D &= -\rho U_1^2 Q(\varepsilon - \varepsilon_w)\\ &= -\rho U_1^2 Q\varepsilon_s\end{aligned}$$

where ε_s is the solid-blockage increment. Noting that

$$Q = \tfrac{1}{2}SC_D$$

there is obtained finally

$$\Delta C_D = -C_D\varepsilon_s$$

which is the result given in Ref. 3.

Note that it was assumed that the displacement surface base area Q is unchanged under constraint. This assumption could be questioned for high-blockage flows where the changes in the wake shape are likely to be significant. Hackett's experience of such flows suggests that this is a significant factor. For attached flows this assumption is reasonable but perhaps ought to be checked for flows approaching separation. It is, however, most unlikely that the wake will always change shape under constraint in such a way that the first term in Eq. (5) is identically zero.

Thus, in summary, we, like Mokry, consider that Hackett's analysis is correct but that it excludes terms that, in the circumstances of attached flow, contribute to the correction to drag for pressure

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gradient. In these circumstances, the present authors would prefer to use the classical correction formula.[3]

References

[1]Mokry, M., "Comment on 'Tunnel-Induced Gradients and Their Effect on Drag,'" *AIAA Journal*, Vol. 36, No. 2, 1998, p. 301.

[2]Hackett, J. E., "Tunnel-Induced Gradients and Their Effect on Drag," *AIAA Journal*, Vol. 34, No. 12, 1996, pp. 2575–2581.

[3]Garner, H. C., Rogers, E. W. E., Acum, W. E. A., and Maskell, E. C., "Subsonic Wind Tunnel Wall Corrections," AGARDograph 109, Oct. 1966, pp. 319–321.

[4]Taylor, C. R., "Some Fundamental Concepts in the Theory of Wind-Tunnel Wall Constraint and Its Applications," Defence Research Agency, DRA/AS/HWA/TR96055/1, 1996.

[5]Woods, L. C., *The Theory of Subsonic Plane Flow*, Cambridge Aeronautical Series, Cambridge Univ. Press, New York, 1961, pp. 16, 17.

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Reply by the Author to P. R. Ashill and C. R. Taylor

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I WOULD like to thank Dr. Ashill and Mr. Taylor for their comments and their demonstration of the linkages between the result of Ref. 1, the classical result of Ref. 2, and the more recent result of Ref. 3. The early part of their comment closely parallels the analysis of Mokry,[4] and the same comments apply.[5] However, their recognition of the importance of the base pressure term in the momentum equation goes to the core of what has become a somewhat controversial issue.

The principal difficulty of the present problem is the tunnel-induced change of the shape of the displacement surface, relative to its unconstrained shape. For example, the far wake displacement (base) area in the tunnel differs from that in free air. Any attempt to force the two to be equal, as when applying the equivalence principle, distorts the relationship between model cross-sectional area and base area, with an unknown effect on drag. The choice of ϵ in Eq. (7) is also an issue. It is asserted here that, if a value of p_c is selected that is appropriate for the working section, e.g., using $\epsilon = \epsilon_w + \epsilon_s$, as in the Ashill–Taylor comment, this will result in an incorrect estimate of base drag. Rather, it is believed that a value of ϵ corresponding to the base location, i.e., $\epsilon = 2\epsilon_w$, should be used. The latter procedure is analogous to correcting the static pressure distribution along a model using a q correction that is a function of x, as described in Ref. 1.

Using $\epsilon = 2\epsilon_w$, Eq. (9) of the Ashill–Taylor comment becomes

$$\Delta D = -\rho U_1^2 Q\left[2\epsilon_w - \tfrac{1}{2}(Q/C)\right]$$

$$= -\rho U_1^2 Q \epsilon_w$$

giving

$$\Delta C_D = -C_D \epsilon_w$$

in agreement with Ref. 1.

Figure 1 extends the corrected normal flat plate drag data in Fig. 7 of Ref. 1 to include results derived using the residual drag expressions of Ashill–Taylor (and Ref. 2) and of Taylor (Ref. 3). Reference 1 uses $\Delta C_D = -C_D \epsilon_w$; Ref. 2 uses $\Delta C_D = -C_D \epsilon_s$; and Ref. 3 uses $\Delta C_D = -C_D(\epsilon_w + \epsilon_s)$. A similar comparison, using a more extensive data set, may be found in Ref. 6. The results speak for themselves.

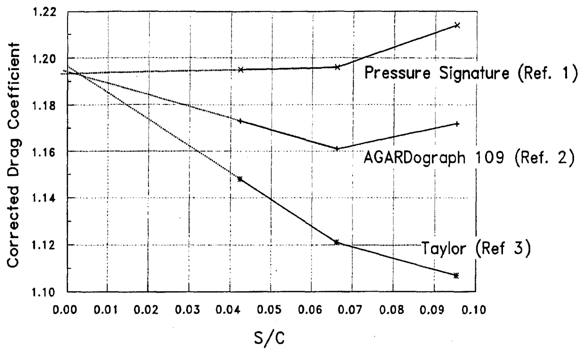

Fig. 1 Drag coefficients for square flat plates using various correction procedures.

References

[1]Hackett, J. E., "Tunnel-Induced Gradients and Their Effect on Drag," *AIAA Journal*, Vol. 34, No. 12, 1996, pp. 2575–2581.

[2]Garner, H. C., Rogers, E. W. E., Acum, W. E. A., and Maskell, E. C., "Subsonic Wind Tunnel Wall Corrections," AGARDograph 109, Oct. 1966, pp. 319–321.

[3]Taylor, C. R., "Some Fundamental Concepts in the Theory of Wind-Tunnel Wall Constraint and Its Applications," Defence Research Agency, DRA/AS/HWA/TR96055/1, Farnborough, England, UK, June 1996.

[4]Mokry, M., "Comment on 'Tunnel-Induced Gradients and Their Effect on Drag,'" *AIAA Journal*, Vol. 36, No. 2, 1998, p. 301.

[5]Hackett, J. E., "Reply by the Author to M. Mokry," *AIAA Journal*, Vol. 36, No. 2, 1998, p. 302.

[6]Ewald, B. (ed.), "Wind Tunnel Wall Corrections," AGARDograph 336, Chap. 6 (manuscript in preparation).

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Comment on "Tunnel-Induced Gradients and Their Effect on Drag"

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I WOULD like to take this opportunity to point out a typographical error and a limitation in the equations for the two-step method proposed by Hackett for the blockage correction of separated flows in a closed-wall wind tunnel. At the same time, I would like to offer some experimental verification of the correct analysis that was developed by Hackett in Ref. 1.

In Ref. 1, Hackett separated Maskell's wake blockage correction into its two components—a dynamic pressure change and an incremental drag adjustment—instead of combining both correction terms into a too-large dynamic pressure correction. The benefit of the split is that drag is better corrected at large blockage and that the other forces and moments are corrected properly. The current *AIAA Journal* paper, Ref. 2, presented a version of the original analysis from Ref. 1 that was linearized to simplify its application. It was found to return a different result from that of Ref. 1. After conversations with Hackett, it appeared that part of the problem was a typographical error in the *AIAA Journal* paper that was not present

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